

Amostrador de Gibbs

2026

Neste problema, $y_1, \dots, y_n$ são valores observados. A distribuição de $(X_1, X_2)^T$ tem função densidade tal que

$$f(x_1, x_2) \propto x_2^{\frac{n}{2}+1} \exp\left(-\frac{x_2}{2} \sum_{i=1}^n (y_i - x_1)^2\right) \exp\left(-\frac{x_1^2}{2}\right) \exp(-x_2),$$

em que $x_1 \in \mathbb{R}$ e $x_2 \in (0, \infty)$; $f(x_1, x_2) = 0$, caso contrário.

Devemos gerar uma amostra da distribuição de $(X_1, X_2)^\top$. Iniciamos obtendo as distribuições condicionais completas. Temos que

$$\begin{aligned} f_2(x_2) &= f(x_2 \mid x_1) \\ &\propto x_2^{\frac{n}{2}+1} \exp\left(-x_2 \left\{1 + \frac{\sum_{i=1}^n (y_i - x_1)^2}{2}\right\}\right) \\ &= x_2^{(\frac{n}{2}+2)-1} \exp\left(-x_2 \left\{1 + \frac{\sum_{i=1}^n (y_i - x_1)^2}{2}\right\}\right). \end{aligned}$$

Portanto,

$$X_2 \mid X_1 = x_1 \sim \text{gama}\left(\text{forma} = \frac{n}{2} + 2, \text{ taxa} = 1 + \frac{\sum_{i=1}^n (y_i - x_1)^2}{2}\right).$$

Também temos que

$$f_1(x_1) \propto \exp\left(-\frac{x_1^2}{2} - \frac{x_2}{2} \sum_{i=1}^n (y_i - x_1)^2\right).$$

Pode ser provado que

$$X_1 \mid X_2 = x_2 \sim \text{normal}\left(\frac{x_2 \sum_{i=1}^n y_i}{1 + nx_2}, \frac{1}{1 + nx_2}\right).$$

```
# Separador decimal: ", "  
options(OutDec = ",")
```

```
# Número de iterações  
M <- 500  
descarte <- 100
```

```
# y1, ..., yn  
y <- c(4.4, 9., 2 -1.1, -1.8, -1.5, 1.7, 7.9, 1.7, 1.0, 0.6, 4.9, 7.1,  
      -1.2, 0.3, -7.9)  
n <- length(y)  
cat("\n n =", n, "\n")
```

```
##  
## n = 15
```

```

forma <- n / 2 + 2
somay <- sum(y)

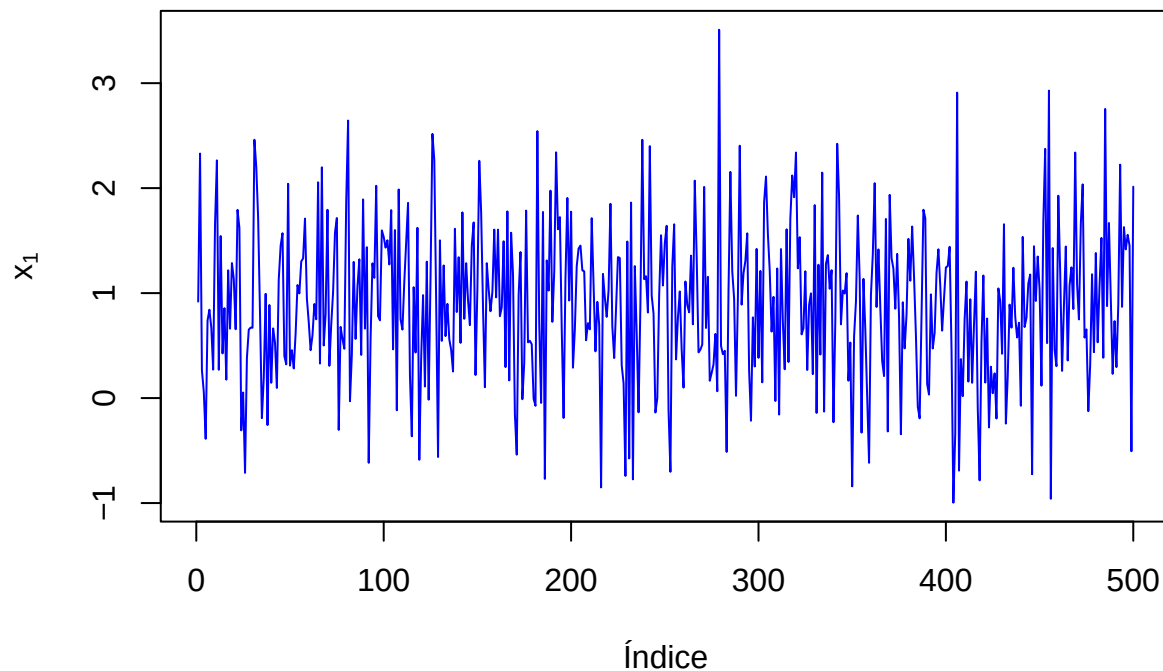
# Valores iniciais
x1 <- x2 <- c()
x1[1] <- -1
x2[1] <- 2.5

set.seed(902)
for (j in 2:(descarte + M)) {
  x2[j] <- rgamma(1, shape = forma, rate = 1 + sum((y - x1[j - 1])^2) / 2)
  denom <- 1 + n * x2[j]
  mux1 <- x2[j] * somay / denom
  sigx1 <- sqrt(1 / denom)
  x1[j] <- rnorm(1, mux1, sigx1)
}

x1 <- x1[-(1:descarte)]
x2 <- x2[-(1:descarte)]

plot(x1, type = "l", xlab = "Índice", ylab = expression(x[1]), col = "blue")

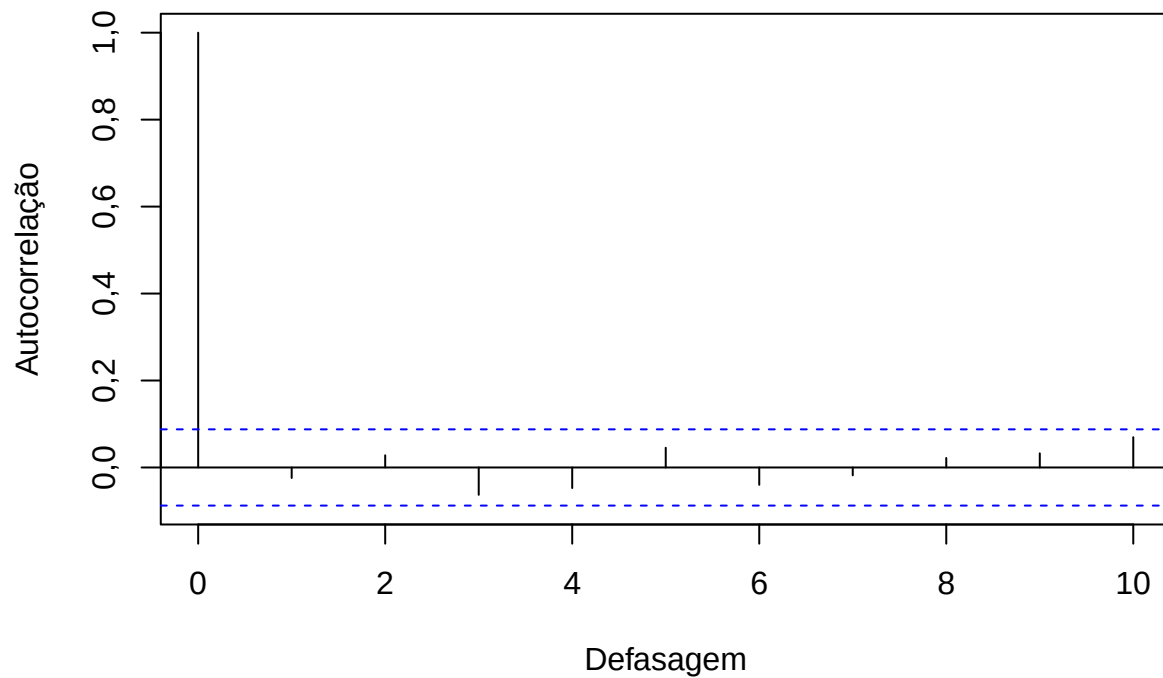
```



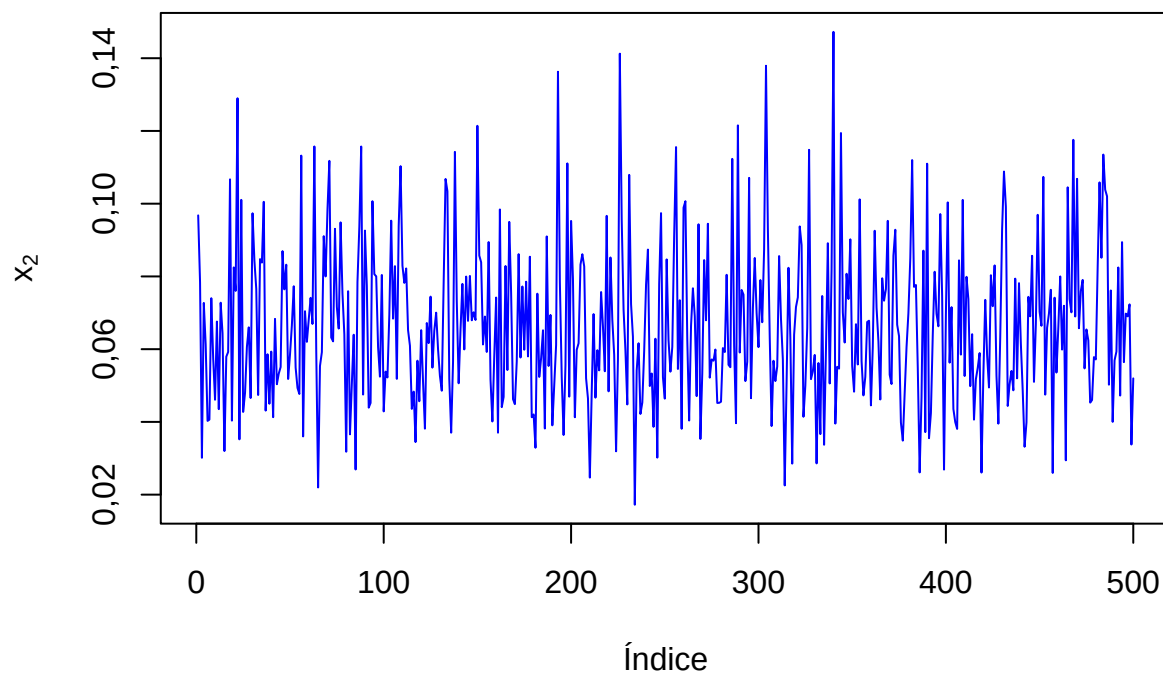
```

acf(x1, lag.max = 10, main = "", xlab = "Defasagem", ylab = "Autocorrelação")

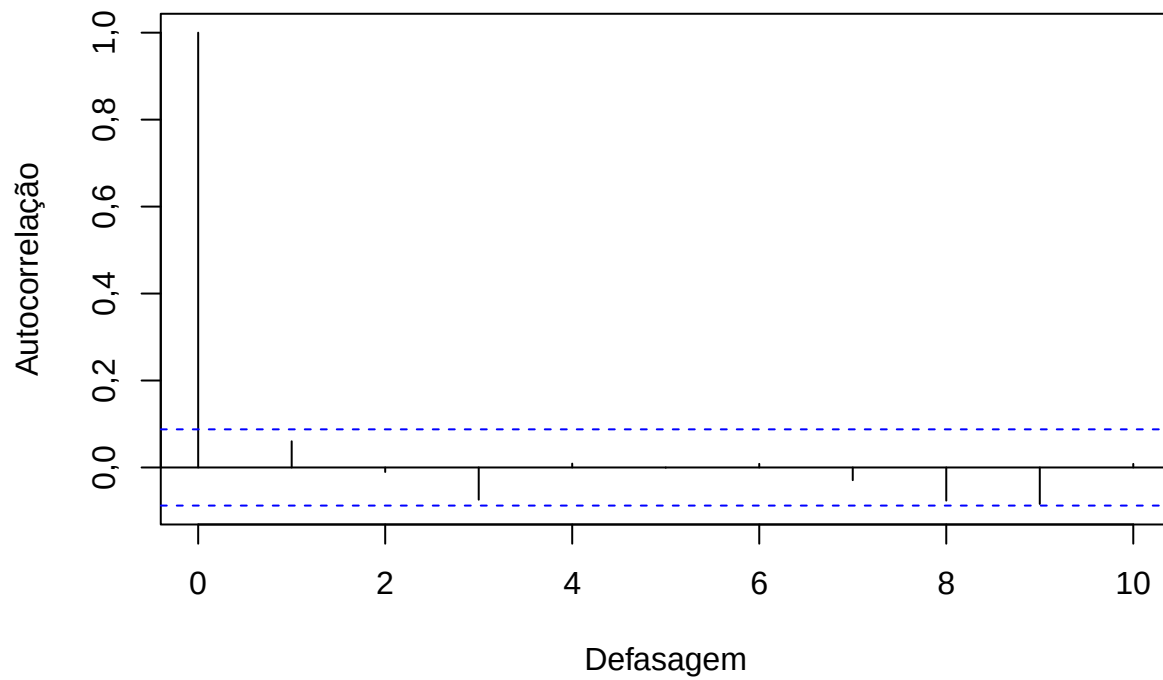
```



```
plot(x2, type = "l", xlab = "Índice", ylab = expression(x[2]), col = "blue")
```



```
acf(x2, lag.max = 10, main = "", xlab = "Defasagem", ylab = "Autocorrelação")
```



```
# (X1, X2)
plot(x1, x2, pch = 20, xlab = expression(x[1]), ylab = expression(x[2]),
     col = "red")
```

